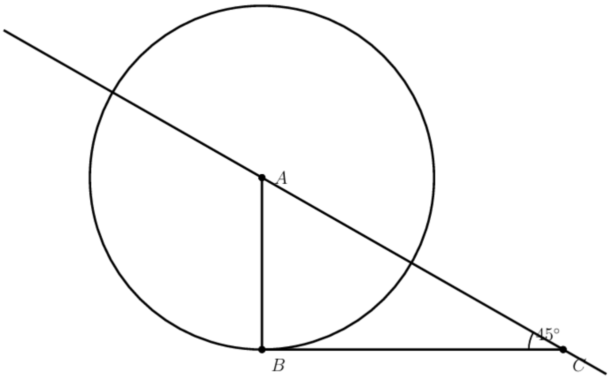
Q1. We are given a circle centred around A with a tangent line BC,



(a) Calculate the angle BAC

(b) The length of BC is 3 now calculate the area of the intersection of the circle and the triangle.

1)

a) Angles sum to 180:

$45 + 90 + x = 180$

therefore $x = 45$

b) length of BC is 3 so

$\frac{AB}{BC} = \tan 45$

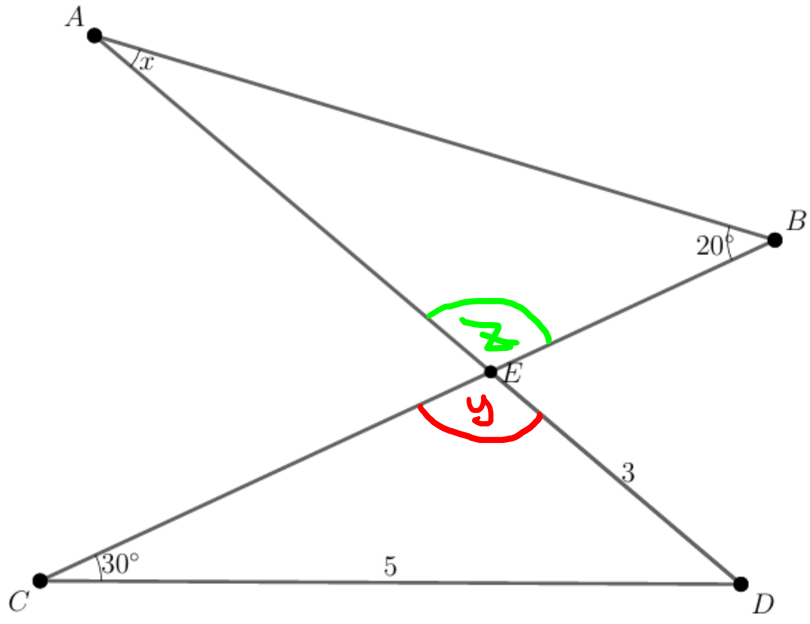
so $AB = 3 \times \tan 45$
$= 3$

Now are of the circle :

$\pi r^2 = \pi (3)^2 = 9\pi$,

So Area of the segment :

$\frac{45}{360} \times 9\pi = \frac{9}{8}\pi$



(a) Calculate the angle of x

use sine rule to work out

y angle,

$$\frac{\sin 30}{3} = \frac{\sin y}{5}$$

$\sin y = \frac{5}{3} \times \sin 30$ then $y = \sin^{-1}\left(\frac{5}{3} \times \sin 30\right)$

$= \sin^{-1}\left(\frac{5}{6}\right)$

$= 56.4$

since z, y are opposite angles $z = y$

So $x + 20 + 56.4 = 180$

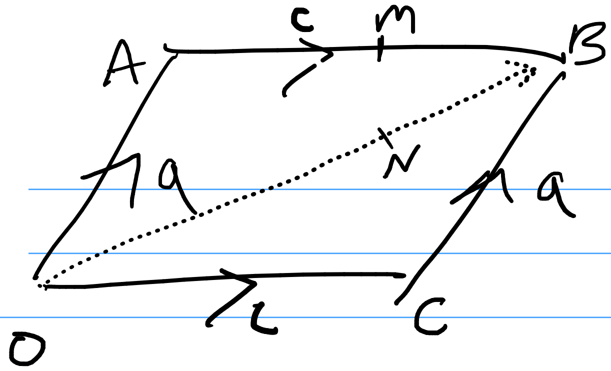
$$\boxed{x = 103.6}$$

Q3. OABC is a parallelogram where $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$.
Point M is the midpoint of AB. Point N lies on OB such that $ON : NB = 2 : 1$.

(a) Find $\overrightarrow{OM}$ in terms of a and c.

(b) Find $\overrightarrow{ON}$ in terms of a and c.

(c) Show that C, M and N are collinear.



a) $\overrightarrow{OM} = a + \frac{1}{2}\overrightarrow{AB}$

$= a + \frac{1}{2}c$

$\overrightarrow{ON} = \frac{2}{3}\overrightarrow{OB} = \frac{2}{3}(a+c)$

b) Need to show that both $\overrightarrow{MN}$ and $\overrightarrow{NC}$ are multiples of $\overrightarrow{MC}$.

$\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC}$

$= \frac{1}{2}c - a$

$\overrightarrow{NC} = \frac{1}{3}\overrightarrow{OB} + \overrightarrow{BC} = \frac{1}{3}(a+c) - a$

$= -\frac{2}{3}a + \frac{1}{3}c$

therefore $\boxed{\frac{2}{3}\overrightarrow{MC} = \overrightarrow{NC}}$

Also $\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN}$

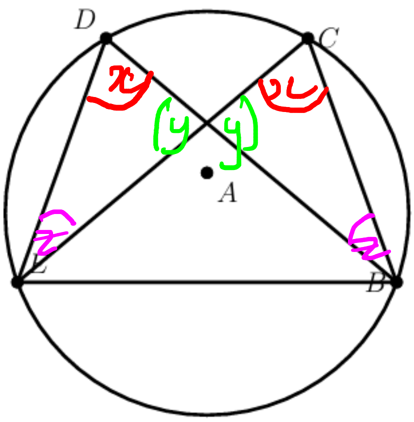
$= \frac{1}{2}c - \frac{1}{3}(a+c)$

$= -\frac{1}{3}a + \frac{1}{6}c = \frac{1}{3}\overrightarrow{MC}$

Therefore:

$$\overrightarrow{MN} = \frac{1}{3}\overrightarrow{MC}$$

Q4. Below is a circle centred at A with two triangle.



(a) Justifying each step explain why the triangle BDE and BCE are similar but not necessarily congruent

(b) What extra information could help us conclude they are congruent.

These angles must be equal since angles in the same segment from the same chord must be equal

These angles must be equal since opposite angles are equal.

The remaining angle must be equal since the other two angles are. To be precise $z = 180 - x - y$.

This is enough to conclude the triangles are similar but not necessarily congruent.

b) since we have three matching angles if one side length matches we can use AAS or ASA to conclude they are congruent.